TABLE I
OUTPUT IMPEDANCE ACROSS THE GRID OF THE REFLEX KLYSTRON

Reactor	R_{sh}	X_{sh}	$R_{sh} \cos \phi$
Cutoff	$2.99 \times 10^5\ \Omega$	$-3.91 \times 10^3\ \Omega$	$3.91 \times 10^3\ \Omega$
Coaxial	$2.75 \times 10^5\ \Omega$	$-19.01 \times 10^3\ \Omega$	$1.89 \times 10^3\ \Omega$

TABLE II
GAIN MEASURED AND CALCULATED

Reactor	Gain (db) Measured	Anode Voltage V_0	Repeller $-V_r$	I_0 (ma)	N	β	Gain (db) Calculated
Cutoff	23.2	280	88	7.7	10.75	0 525	22 71
Coaxial	28.0	304	268	2.5	6.75	0 55	29.1

Note: $\beta = \sin \dfrac{3170\, dg}{2\lambda\sqrt{V_0}} \Big/ \dfrac{3170\, dg}{2\lambda\sqrt{V_0}}, \quad N = 4fl \Big/ \sqrt{\dfrac{m}{2e}}\, V_0/(V_0 + V_r)$

$1 = 3.44 \times 10^{-3}$ m,
Gap-distance $dg = 0.61 \times 10^{-3}$ m.

where I_s' and K_s' are the modified Bessel functions and

$$\sigma \equiv \sqrt{\left(\frac{\pi}{4s}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}. \quad (6)$$

This cutoff reactor gives the same positive reactance for extremely small distance of the shorting plunger setting in comparison with the coaxial reactor. The cutoff reactor is mechanically simpler than the coaxial type.

The numerical circuit constants used to obtain the values of R_{sh} and X_{sh} are shown in Figs. 2–4. The dimensions are based on measurement of the actual circuit used. The constants of the klystron cavity C_P, L_P, L_s, and M were calculated by Fujisawa's method.[5,6] The impedance Z_{g1} and Z_{g2} in Fig. 2 were calculated by Tanaka's method.[7] The reactance of the antenna, X_0, was calculated assuming it was a uniform cylindrical conductor. The computed results of R_{sh} and X_{sh} are given in Table I, where $\cos \phi$ is the power factor of the circuit. These values were checked by the experiment in the following way.

The gain of the reflex klystron amplifier can be calculated by the following equation[8] based on the regenerative action of the electron beam.

$$A = \frac{V_0}{V_0 - I_0\beta^2 R_{sh}\pi N \cos\phi} \quad (7)$$

where

A = gain,
V_0 = anode voltage,
I_0 = effective electron beam current,
β = beam coupling coefficient,
N = number of electron transit time cycles in the repeller space.

The calculated results are listed in Table II with some measured data such as anode and repeller voltage, effective current, and dimensions of the tube. As shown in this table, the calculated gains are in good agreement with the measured gains.

KORYU ISHII
Dept. of Elec. Engrg.
Marquette University
Milwaukee, Wis.

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[7] S. Tanaka, "A broad band coaxial to wave-guide junction," *J. Inst. Elec. Commun. Engrs. Japan*, vol. 37, pp. 172–176; March, 1954.
[8] T. Okabe, "Microwave Amplification by the Use of Reflex Klystrons," Rept. of Microwave Res. Committee in Japan, Tokyo; June–July, 1952.

Some Comments on the Method of Kyhl*

The purpose of this letter is not to criticize the philosophy of Kyhl's[1] method nor even to engage in a debate as to whether his method is, or is not, more useful than ours.[2,3] We find his proposals both interesting and meaningful. First, we would like to call attention to two errors in his letter and then to show the close relation of his method to ours.

Let us consider his method. He proposes that a "Γ" $= 1/\Gamma$ be used when $|\Gamma| > 1$. This, he claims, will produce his "double SMITH-HTIMS chart." This is in error because, if $|\Gamma| > 1$, the original point in the extended Standard Smith chart[3] is without the unit circle, and the inversion type operation (with or without the minus sign) within the unit circle; hence, the "HTIMS" part of the chart will coincide with the regular Smith chart.

To obtain Kyhl's "double SMITH-HTIMS chart," we suggest the following construction. An inversion in the unit circle, followed by a symmetry[4] (reflection) with respect to the line $\Gamma = 1$. This can be written analytically as "Γ" $= -\Gamma'^* + 2$, where $\Gamma'^* = 1/\Gamma^*$.

We next consider the statement that our method is not analytic. By this statement, we presume that Kyhl means that the modified β and the modified β^{-1} transformations, or the Darboux transformation[5] as shown in footnote 2, is presented in its natural graphical form. The following is the transformation in analytic form:

$$[\Gamma_1 = 1/\Gamma^*, \quad \ln\,[O\Gamma', AB] = 2 \ln\,[O\Gamma_1, AB],$$

$$\Gamma' = \frac{2\Gamma_1}{1 + |\Gamma_1|^2} = \frac{2\Gamma}{1 + |\Gamma|^2}$$

where A and B are the two points on the unit circle, intersected by the straight line, connecting the center (O) and the inverse point (Γ_1). The brackets indicate the cross ratio of the four points in question. The final analytic form shows the invariance of the final point with respect to inversion in the unit circle, which was proven geometrically.[2] It is noted that the above equation is essentially the same as that stated by Deschamps[6] for the β transformation.

In comparing our results with Kyhl's, we shall use his method with our modification. It is conceded that there are other ways to correct the proposed method. The first step of inversion is the same in both procedures. The difference occurs in the second step. Our transformation (which is of course the Deschamps'[7] β and β^{-1} transformation) is best considered as finding the non-Euclidean bisector of a line segment.[6,8] The second step in Kyhl's procedure, the reflection, is an involution.[9] The inversion[9] (also an involution), followed by the reflection, is a graphical way of performing a nonloxodromic bilinear transformation. The unit circle is the isometric circle of the equivalent bilinear transformation, which is "Γ" $= (2\Gamma - 1)/\Gamma$. This result also could have been obtained analytically, instead of using the geometrical interpretation. In summary, it is noted that Kyhl's and our results are quite similar in form, both analytically and geometrically, but differ mostly in the final presentation.

L. J. KAPLAN
D. J. R. STOCK
Elec. Engrg. Dept.
New York University
New York, N. Y.

* Received by the PGMTT, June 16, 1960.
[1] R. L. Kyhl, "Plotting impedance with negative resistive components," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, p. 337; May, 1960.
[2] D. J. R. Stock and L. J. Kaplan, "An extension of the reflection coefficient chart to include active networks," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 298–299; April, 1959.
[3] L. J. Kaplan and D. J. R. Stock, "The representation of impedances with negative real parts in the projective chart," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, p. 475; October, 1959.
[4] R. Deaux, "Introduction to the Geometry of Complex Numbers," F. Ungar Publishing Co., New York, N. Y.; 1956.
[5] E. F. Bolinder, "Theory of noisy two-port networks," *Tech. Rep.* 344, and personal correspondence, M.I.T. Res. Lab. for Electronics, Cambridge, Mass.
[6] G. A. Deschamps, "A Hyperbolic Protractor for Microwave Impedance Measurement," Fed. Telecommun. Lab., Nutley, N. J., 1953.
[7] G. A. Deschamps, "Determination of reflection coefficients and insertion loss at a wave-guide junction," *J. Appl. Phys.*, vol. 24, pp. 1046–1050; August, 1953.
[8] L. J. Kaplan and D. J. R. Stock, "Non-Euclidean Geometric Representations for Microwave Networks," New York University, College of Engrg., Tech. Note 400-3, pt. 2; October, 1959.
[9] E. F. Bolinder, "Impedance and Power Transformations by the Isometric Circle Method and Non-Euclidean Hyperbolic Geometry," M.I.T. Res. Lab. for Electronics, Cambridge, Mass., Tech. Rept. 312; June, 1957.

Reply by R. L. Kyhl

I have no disagreement with the comments of L. J. Kaplan and D. J. R. Stock. I was thinking of plotting the two parts of the chart from different origins. My chief interest was in the type of graphical display chosen.

While the issue was at the press, a similar use of a double chart was published elsewhere.[10]

[10] R. M. Steere, "Novel applications of the Smith chart," *Microwave J.*, vol. 3, pp. 97–100; March, 1960.

Scattering Matrix for an N-Port Power-Divider Junction*

Introduction

During the course of an investigation of a data-processing technique yielding effectively reduced sidelobes and beamwidth for a microwave radar antenna, the need arose for multiport power dividers. In order to avoid an undesirable decrease in the signal-to-noise ratio, it was necessary that these dividers waste no power. Consequently, a scattering matrix was sought which would have the obvious requirement that there be no wave reflected in the input port and which would allow the power to be divided into arbitrary but fixed relative parts.

It should be noted that there now exist two methods[1] for synthesizing an n-port junction at a single frequency directly from the normalized scattering matrix, without use of the associated impedance matrix.

The Scattering Matrix

A reciprocal, lossless junction can be represented by a symmetric, unitary scattering matrix $\mathbf{S}'$. It is sufficient to consider a real matrix first without losing generality, because the matrix $\mathbf{S}'$ for the general case (including phase shifts) can be derived from the real case by a simple transformation.[2]

The purpose of this paper is to find the scattering matrix for an n-port junction, such that when a wave is fed into, say, port one, there will be no reflected wave in that port, and such that the amplitude of the wave transmitted to port k is equal to a given x_k. Analytically expressed, this requires that

$$x_k = \sum_i S_{ki}\delta_{1i} = S_{k1}, \qquad (1)$$

where the x_k are subject to the restriction

$$x_1 = 0 \quad \text{and} \quad \sum_{k=2}^{n} x_k^2 = 1. \qquad (2)$$

* Received by the PGMTT, July 15, 1960.

[1] D. C. Youla, "Direct single frequency synthesis from a prescribed scattering matrix," IRE Trans. on Circuit Theory, vol. CT-6, pp. 340–344; December, 1959.

[2] "Reference Data for Radio Engineers," American Book—Stratford Press, Inc., New York, N. Y., 4th ed.; 1956.

Symmetry requires that

$$S_{ij} = S_{ji}, \qquad (3)$$

and unitarity, which reduces to orthogonality for the real case considered here, requires that

$$\sum_{k=1}^{n} (\tilde{S})_{ik} S_{kj} = \sum_{k=1}^{n} S_{ik} S_{kj} = \delta_{ij} \qquad (4)$$

[where use has been made of (3)].

the matrix $\mathbf{S}$ satisfying (1), (3), and (4) may be obtained as follows:

$$\left.\begin{aligned} &\text{A. } S_{11} = 0, \\ &\text{B. } S_{kk} = x_k^2 - 1 \quad \text{for } k > 1, \\ &\text{C. } S_{ij} = S_{ji} = x_i x_j \text{ for } i \neq j, i > 1, j > 1, \\ &\text{D. } S_{1i} = S_{i1} = x_i \quad \text{for } i > 1. \end{aligned}\right\} (5)$$

To prove that (5) is the required solution, it is only necessary to verify that it satisfies (1), (3), and (4). From A and D, it is evident that (1) is satisfied, and from C and D it is evident that (3) is satisfied. To verify that (4) is also satisfied, it is necessary to consider separately various possible values of i and j, because of the special nature of the various S_{ij}.

1) $i=j=1$.

Using (2) and the rules given in (5),

$$\sum_{k=1}^{n} S_{1k}S_{k1} = \sum_{k=2}^{n} x_k^2 = 1,$$

which satisfies (4).

2) $i=j\neq 1$.

Proceeding as above,

$$\begin{aligned} \sum_{k=1}^{n} S_{ik}S_{ki} &= S_{i1}^2 + S_{ii}^2 + \sum_{k\neq 1,i} S_{ik}S_{ki} \\ &= x_i^2 + (x_i^2 - 1)^2 + x_i^2 \sum_{k\neq i} x_k^2. \end{aligned}$$

According to (2), the above sum on k is $(1-x_i^2)$ and hence the right side reduces to one as required.

3) $i\neq j$.

$$\begin{aligned} \sum_{k=1}^{n} S_{ik}S_{kj} = S_{i1}S_{1j} + S_{ii}S_{ij} + S_{ij}S_{jj} \\ + \sum_{k\neq 1,i,j} S_{ik}S_{kj}. \end{aligned} \qquad (6)$$

It is necessary to consider separately the case $i=1$ (or $j=1$) and $i\neq 1\neq j$.

a) $i=1$.

For this case, the above expression rereduces to

$$\sum_{k=1}^{n} S_{1k}S_{kj} = x_j(x_j^2 - 1) + x_j \sum_{k\neq j} x_k^2 = 0.$$

The case $j=1$ is essentially the same as that above and therefore (4) is satisfied in both of these cases.

b) $i\neq 1\neq j$.

For this case, (6) becomes

$$\begin{aligned} \sum_{k=1}^{n} S_{ik}S_{kj} = x_i x_j + x_i x_j (x_i^2 + x_j^2 - 2) \\ + x_i x_j \sum_{k\neq i,j} x_k^2. \end{aligned}$$

When use is made of (2), the above equation reduces to zero as required.

Conclusion

It has been shown that it is theoretically possible to provide a junction which will divide an input wave into many output waves of arbitrary but fixed relative amplitudes. The arbitrariness of the power division means that one may choose any ratios for the output waves and have no reflected wave in the input port. Once such a divider is constructed, however, the ratio of the output waves is fixed.

O. R. Price
Hughes Res. Labs.
A Division of Hughes Aircraft Co.
Malibu, Calif.
M. Leichter
Ground Systems Group
A Division of Hughes Aircraft Co.
Fullerton, Calif.

Lossy Resonant Slot Coupling*

The paper by Allen and Kino[1] suggests a novel method of combating troublesome cutoff oscillations in periodic slow-wave structures. The idea is to couple loss periodically into the system through slots which are resonant at the center of the (narrow) oscillation range. The high Q of the slots will effectively decouple the loss in the operating range of the pass band.

To develop this idea, we start with Allen and Kino's (13) for the voltage $\phi(x)$ along the slot in terms of the tangential $\hat{H}$ field along the slot. With respect to their Fig. 2 coordinates, shown in Fig. 1 below, we can

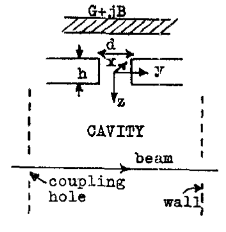

Fig. 1—A lossy slot in a cavity wall. Voltage $\phi(x)$ exists across the slot gap, d.

write the transmission line equation for the slot voltage as

$$\begin{aligned} (\partial^2/\partial x^2 + k^2)\phi(x) \\ = -j\omega L_0[(I_c\hat{H}_{cx})_+ - \hat{H}_{x-}] \end{aligned} \quad (1)$$

where the tangential magnetic field is $(I_c\hat{H}_{cx})_+$ on the $+z$- or cavity side of the slot and is $\hat{H}_{x-}$ on the $-z$-side. Eq. (1) can be derived rigorously for a TEM slot mode. The caret denotes a total field, and we have split the cavity field into an amplitude I_c and a vector field pattern $\hat{H}_c$ for equivalent circuit purposes; this notation differs from that of Allen and Kino. L_0 is the slot inductance per unit length in the x-direction.

Let us introduce the lossy susceptance $G+jB$ by saying that the average voltage

* Received by the PGMTT, July 20, 1960.

[1] M. A. Allen and G. S. Kino, "On the theory of strongly coupled cavity chains," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 362–372; May, 1960.